

$Z_c(3900)$ as a Four-Quark State

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Abstract

Using the method of QCD Sum Rules, we derive the correlator Π^Z for a state consisting of two charm quarks and two light quarks, $c\bar{d}\bar{c}u$, and carry out a Borel transform to find $\Pi^Z(M_B)$. From this we find the solution that $M_B \simeq 3.9 \pm .2$ GeV, showing that the $Z_c(3900)$ is a tetra-quark state.

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1 Introduction

Recently e^+e^- collision experiments by BESIII[1] and Belle[2] Collaborations have found a state at about 3,900 MeV, called the $Z_c(3900)$ [1], that might be a four quark state, $|c\bar{d}\bar{c}u\rangle$ [3]

For many years states in the energy region of 3,900-4,500 MeV, near the $D\bar{D}$ threshold, there have been found possible tetra-quark states. See, e.g., Ref[4], for a theoretical study of the X(3872) as a charm tetra-quark about one decade ago. Recently, there have been studies of the $Z_c(3900)$ as a $\bar{D}D^*$ molecular state[5, 6], vector/axial vector Charmonium state[7] using the method of QCD Sum Rules as in the present study. See these references for references to earlier publications, as well as a review of New Charmonium States via QCD Sum Rules[8]

In our study of the $Z_c(3900)$ as possibly a $|c\bar{d}\bar{c}u\rangle$ state we use the method of QCD sum rules[9]. Our approach differs from earlier studies[5, 6] in that our correlator corresponds to a four-quark (tetra-quark) rather than a $\bar{D}D^*$ molecular state. First we find the correlator in momentum space, and then as a function of the Borel mass, M_B , and see if it has a minimum near the value of 3.9 GeV, similar to our study of heavy quark hybrid meson states[10].

In Section II we briefly review the method of QCD sum rules, and derive the correlator for our four-quark model. In Section III we find the correlator as a function of the Borel mass in the region near 4,000 MeV; and then from a plot find the minimum value of M_B . In Section IV we discuss the results and conclusions.

2 The $|cd\bar{c}u\rangle$ state and QCD Sum Rules

The method of QCD sum rules[9] for finding the mass of a state A starts with the correlator,

$$\Pi^A(x) = \langle 0|T[J_A(x)J_A(0)]|0\rangle, \quad (1)$$

with $|0\rangle$ the vacuum state and the current $J_A(x)$ creating the states with quantum numbers A:

$$J_A(x)|0\rangle = c_A|A\rangle + \sum_n c_n|n; A\rangle, \quad (2)$$

where $|A\rangle$ is the lowest energy state with quantum numbers A, and the states $|n; A\rangle$ are higher energy states with the A quantum numbers, which we refer to as the continuum. One then carries out a Borel transform to reduce the importance of the continuum and higher order diagrams.

For our theory of the $Z_c(3900)$ as a tetra-quark state $|cd\bar{c}u\rangle$, we use the current J_{Z_c} , which creates a $J^{PC} = 1^{+-}$ tetra-quark state:

$$J_{Z_c} = \bar{d}\gamma_5 c \bar{c}\gamma_5 u, \quad (3)$$

with the correlator

$$\Pi^Z(x) = \langle 0|T[J_{Z_c}(x)J_{Z_c}(0)]|0\rangle. \quad (4)$$

Note that as was emphasized by J-R. Zhang in the Summary of Ref[5] the current and correlator used for a QCD sum rule study of the $Z_c(3900)$ as a $\bar{D}D^*$ molecular state, with a current and correlator given by

$$\begin{aligned} J_{\bar{D}D^*}^\mu &= (\bar{Q}_a i\gamma_5 q_a)(\bar{q}_b \gamma^\mu Q_b) \\ \Pi_{\bar{D}D^*}^{\mu\nu} &= i \langle 0|T[J_{\bar{D}D^*}^\mu(x)J_{\bar{D}D^*}^{\nu+}(0)]|0\rangle, \end{aligned} \quad (5)$$

with Q, q charm, light (u, d) quarks and a, b color indices, is just one possible theoretical interpretation of the $Z_c(3900)$ state, and is not the same as a tetra-quark state. We emphasize that with our current, Eq(3) and correlator, Eq(4), we are using a QCD sum rule to explore the possibility that the $Z_c(3900)$ is a tetra-quark state, which is not completely orthogonal to but is quite different from a $\bar{D}D^*$ molecular state, by finding the mass using the tetra-quark correlator.

Using Wick's Theorem to express $\Pi^Z(x)$ in terms of the quark propagators, and taking the Fourier transform, one obtains the correlator in momentum space:

$$\Pi^Z(p) = \int \frac{d^4 k_1 d^4 k_2 d^4 k_3}{((2\pi)^4)^3} Tr[S_d(k_1)\gamma_5 S_c(k_2)\gamma_5] Tr[S_c(k_3)\gamma_5 S_u(p+k_1-k_3+k_3)\gamma_5], \quad (6)$$

with $S_q(k)$ a quark propagator,

$$S_q(k) = \frac{\not{k} + m_q}{k^2 - m_q^2}, \quad (7)$$

where $\not{k} = \sum_\alpha \gamma^\alpha k_\alpha$, with γ^α a Dirac matrix.

The correlator is illustrated in Fig. 1

Note that higher order terms are very small, as pointed out in Ref[6].

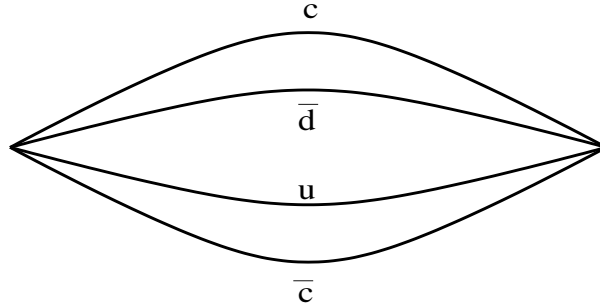


Figure 1: c, \bar{c} are charm, anticharm quarks. u, \bar{d} are up, antidownquarks

Finally, we carry out a Borel Transform[9]

$$\mathcal{B}\Pi^Z(p) = \Pi^Z(M_B) \quad (8)$$

3 The Correlator $\Pi^Z(p) \rightarrow \Pi^Z(M_B)$

From Eqs(6,7), carrying out the traces, one finds

$$\frac{\Pi^Z(p)}{16} = \int \frac{d^4 k_1 d^4 k_2 d^4 k_3}{((2\pi)^4)^3} \frac{mM(mM - k_1 \cdot k_2) + k_3 \cdot (p + k_1 - k_2 + k_3)(k_1 \cdot k_2 - mM)}{(k_1^2 - m^2)(k_2^2 - M^2)(k_3^2 - M^2)((p + k_1 - k_2 + k_3)^2 - m^2)} \quad (9)$$

Since the $k_1 \cdot k_2$ term vanishes via the momentum integrals and the $k_3 \cdot (p + k_1 - k_2 + k_3)$ term vanishes via the Borel transform, one needs to evaluate two terms;

$$I_1(p) = 16 \int \frac{d^4 k_1 d^4 k_2 d^4 k_3}{((2\pi)^4)^3} \frac{m^2 M^2}{(k_1^2 - m^2)(k_2^2 - M^2)(k_3^2 - M^2)((p + k_1 - k_2 + k_3)^2 - m^2)} \quad (10)$$

$$I_4(p) = 16 \int \frac{d^4 k_1 d^4 k_2 d^4 k_3}{((2\pi)^4)^3} \frac{k_1 \cdot k_2 k_3 \cdot (p + k_1 - k_2 + k_3)}{(k_1^2 - m^2)(k_2^2 - M^2)(k_3^2 - M^2)((p + k_1 - k_2 + k_3)^2 - m^2)} \quad (11)$$

with $M = M(\text{charm quark}) \simeq 1.5 \text{ GeV}$ and $m = m(u, d \text{ quark}) \simeq 4 \text{ MeV}$.

For the evaluation of Eqs(10,11) one uses

$$\begin{aligned} I_H(p) &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M^2)((p-k)^2 - M^2)} \\ &= \frac{1}{(4\pi)^2} \left(\frac{5}{2} p^2 - 4M^2 \right) \int_0^1 d\alpha \frac{1}{\alpha(1-\alpha)p^2 - M^2}, \end{aligned} \quad (12)$$

and

$$\begin{aligned} I_{Hh}(p) &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M^2)((p-k)^2 - m^2)} \\ &= \frac{1}{(4\pi)^2} \left[\left(-\frac{(m^2 - M^2 + p^2)^2}{2p^2} + 2m^2 \right) \int_0^1 d\alpha \frac{1}{\alpha(1-\alpha)p^2 - (1-\alpha)m^2 - \alpha M^2} \right] \\ &\quad + \frac{5}{2} - \frac{3M^2 - m^2}{2p^2} \ln(M^2/m^2). \end{aligned} \quad (13)$$

Carrying out the integrals in Eqs(10,11), one finds

$$\begin{aligned} I1(p) &= \frac{16m^2M^2}{(4\pi)^6} \int_0^1 d\alpha \frac{5(1 - 0.8\alpha(1-\alpha))}{2(\alpha(1-\alpha))^2} \\ &\quad \left[\frac{m^2 - 3M^2\alpha(1-\alpha)}{2\alpha(1-\alpha)} \ln(M^2/m^2) I_{Hh}(p)_{m=0} + \frac{m^2 + M^2\alpha(1-\alpha)}{\beta(1-\beta)\alpha(1-\alpha)} I_{Hh}(p)_{ma^2} \right. \\ &\quad \left. - \left(\frac{m^2}{2(\beta(1-\beta))^2} + \frac{m^2 + \alpha(1-\alpha)M^2}{2m^2\alpha(1-\alpha)} \right) I_{Hh}(p)_{mb^2} + \frac{m^2 + \alpha(1-\alpha)M^2}{2m^2\alpha(1-\alpha)} I_{Hh}(p)_{m=0} \right], \end{aligned} \quad (14)$$

with $ma^2 = (1-\beta)m_1^2 + \beta M^2$ and $mb^2 = m^2/(\beta(1-\beta))^2$; and

$$\begin{aligned} I4(p) &= -\frac{32}{(4\pi)^2} m^2 \int_0^1 \frac{d\alpha}{\alpha(1-\alpha)^2} (5\alpha(1-\alpha) - 7/2) \int_0^1 \frac{d\gamma}{1-\gamma} \\ &\quad \left[M^2 \frac{5m^2 - 7\bar{m}_2^2/4}{\gamma} I_{Hh}(p)_{\bar{m}_2^2} + (p^2 - m^2)(m_4^2 - m_3^2)(2 - 3\gamma) I_{Hh}(p)_{m_4^2} \right], \end{aligned} \quad (15)$$

with $m_3^2 = (M^2 + (1-2\gamma)m_1^2)/(\gamma(1-\gamma))$, $m_4^2 = (M^2 + (\gamma-1)m_1^2)/(\gamma(1-\gamma))$, and $\bar{m}_2^2 = m^2/(\alpha(1-\alpha))$.

Taking the Borel transform one obtains (after removing common factors)

$$\begin{aligned} \mathcal{B}I1(p) &= -5.0 \int_0^1 d\alpha \int_0^1 d\beta (1 - 0.8\alpha(1-\alpha)) \left[((m^2 - 3\alpha(1-\alpha)M^2) \ln(M^2/m^2) \right. \\ &\quad \left. + (m^2 + \alpha(1-\alpha)M^2)/(2(1-\beta)m^2 - \beta\alpha(1-\alpha)M^2)) \right. \\ &\quad \left. (M^2(1 - \frac{\alpha}{2} - \frac{\beta(1-\beta)}{2}) e^{-\alpha M^2/\beta(1-\beta)M_B^2} \right. \\ &\quad \left. + \int_0^1 d\gamma (1/2) \left[\frac{(1+\beta)m^2 + (1-\alpha)(\alpha+\beta)M^2}{M^2} - \frac{(m^2 + \alpha(1-\alpha)M^2)\alpha(1-\alpha)}{(1-\beta)m^2 - \beta\alpha(1-\alpha)M^2} \right] \right. \\ &\quad \left. (3m^2 - M^2 + ((1-\beta)m^2 - \alpha(1-\alpha)M^2)/2) + \frac{(m^2 - M^2)\gamma(1-\gamma)}{2((1-\beta)m^2 - \alpha(1-\alpha)M^2)} \right] \\ &\quad e^{-((1-\beta)m^2 - \alpha(1-\alpha)M^2)/[\alpha(1-\alpha)\gamma(1-\gamma)M_B^2]} \end{aligned} \quad (16)$$

$$\begin{aligned}
\mathcal{BI4}(p) = & - \int_0^1 d\alpha \int_0^1 d\gamma \int_0^1 (5\alpha(1-\alpha) - 7/2)[(5\alpha(1-\alpha)\gamma(1-\gamma) - 7/4)m^2 \\
& (m^2 - M^2\alpha(1-\alpha)\gamma(1-\gamma) + \frac{((m^2 - M^2\alpha(1-\alpha)\gamma(1-\gamma))^2\lambda(1-\lambda)}{2((1-\lambda)m^2 + \lambda\alpha(1-\alpha)\gamma(1-\gamma)M^2)} \\
& + \frac{1}{2}((1-\lambda)m^2 - \lambda\alpha(1-\alpha)\gamma(1-\gamma)M^2)) \\
& e^{-((1-\lambda)m^2 + \lambda\alpha(1-\alpha)\gamma(1-\gamma)M^2)/\alpha(1-\alpha)\gamma(1-\gamma)\lambda(1-\lambda)M_B^2} \\
& - ((3(\gamma-1)m^2 + \alpha(1-\alpha)(1-\gamma(1-\gamma)) - \frac{1}{2}\bar{M}^2)\bar{M}^2 - \frac{1}{2}((\gamma-1)m^2 + \\
& \alpha(1-\alpha)(1-\gamma(1-\gamma))M^2)^2 e^{-\bar{M}^2/[\alpha(1-\alpha)\gamma(1-\gamma)\lambda(1-\lambda)M_B^2]} \quad (17)
\end{aligned}$$

$$B\Pi^Z(p)(p) = \Pi^Z(M_B) = \mathcal{BI1}(p) + \mathcal{BI4}(p) . \quad (18)$$

The correlator as a function of the Borel mass, $\Pi^Z(M_B)$ is shown in Fig. 2 below. Note that the result for the mass is given by M_B at the minimum in the plot of $\Pi^Z(M_B)$, and the error by the shape of the plot near the minimum[9], as is discussed in detail in Ref[10].

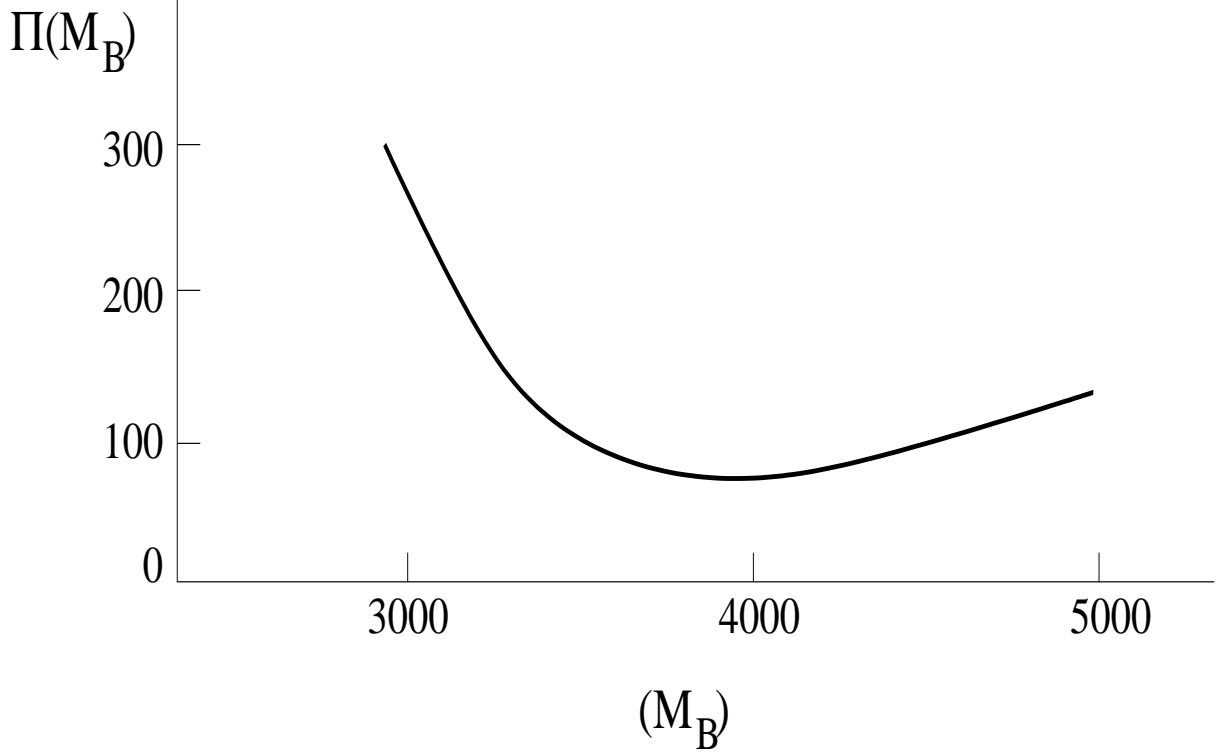


Figure 2: $\Pi(M_B)$ is the 4-quark correlator, a function of the Borel Mass M_B , in units of MeV

4 Results and Conclusions

From Figure 2, the mass of the $|cd\bar{c}u\rangle$ state is 3900 ± 200 MeV, in agreement with the state found in the recent BESIII[1] and Belle[2] experiments. From this we conclude that the conjecture of these two collaborations is correct, that the $Z_c(3900)$ is a tetra-quark state. For decades experimentalists and theorists have attempted to find tetra-quark states, so this is a very important discovery.

Our plans for future research include estimates of the decay probabilities of our tetra-quark theory of the $Z_c(3900)$ to compare with experimental results, which requires a three-point correlator.

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